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THE POLYHEDRON AS A SWITCHING CONCEPT

64-5

TECHNICAL DOCUMENTARY REPORT NO. ESD-TDR-63-439

SEPTEMBER 1963

P. G. Edwards

Prepared for
DIRECTORATE OF ANALYSES
ELECTRONIC SYSTEMS DIVISION
AIR FORCE SYSTEMS COMMAND
UNITED STATES AIR FORCE
L. G. Hanscom Field, Bedford, Massachusetts

Project 600

Prepared by

THE MITRE CORPORATION
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ABSTRACT

This report is a preliminary evaluation of two of the regular polyhedra, the icosahedron and the dodecahedron, and modification thereof, as possible configurations for a survivable switching network concept, involving two interconnected rings of switching points.

THE POLYHEDRON AS A SWITCHING CONCEPT

The idea that examination of the "regular polyhedra" might yield something useful in switching concepts was generated both from a long-standing interest in these figures, and a conviction that virtue is likely to reside in symmetry. While the present thinking in redundancy survivability through switching is concerned principally with the ZI (planar arrangement), the solid figures are adapted to global switching concepts. A possible way of modifying the basic figures is by introducing diagonals which pass through the solid figure.

There are only five regular (non re-entrant) polyhedra. These are:

(a) tetrahedron,

(b) double pyramid-octahedron,

(c) cube,

(d) icosahedron, and

(e) dodecahedron

Euler's theorem relates the number of vertices, edges, and faces as follows

$$V - E + F = 2$$

If these designations are used, and if A, the number of edges terminating at a vertex, and C, the number of sides of each face are used, Table 1 can be derived.[1]

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[1] "Contemporary Geometry," André Delachat. Translated from the French by Howard G. Bergmann. (New York: Dover Publications, Inc.)

TABLE 1

	A	C	V	E	F
Tetrahedron	3	3	4	6	4
Octahedron	4	3	8	12	6
Cube	3	4	6	12	8
Icosahedron	5	3	12	30	20
Dodecahedron	3	5	20	30	12

These numbers are further related:

$$VA = 2E - F^{C}$$

$$\frac{1}{E} = \frac{1}{A} + \frac{1}{C} - \frac{1}{2}$$

Except for reference, the manipulation of these formulas is omitted to prove that these are, in fact, the only possible regular, nonre-entrant polyhedra.

The icosahedron and the dodecahedron are of particular interest because, from a switching standpoint, they contain a reasonable number of vertices (switching points) and number of edges (connecting circuits) terminating at a vertex. These cases will be examined in detail for desirable modifications to improve redundancy by attempting to count the variety of one-link, two-link, three-link, and four-link connections which are possible. Because of human unreliability in counting the paths, errors of duplication and/or omission can easily occur. In addition, to check every switching point against every other switching point becomes, for an individual, a monumental effort. Consequently, the numbers given are indicative but not guaranteed. Detailed tables of the paths counted are included so that errors can be found and corrected. What is needed, assuming the approach is considered important enough to justify

further analysis, is a series of computer runs, a theoretical analysis to provide the possible number of connections without necessarily indicating which particular connections are involved, and/or a KILLCOM-type study to yield a survivability index.

At this point, note that the number of paths given as possible does not take into account the practicability of such paths from the standpoints of fruitless hunting, digital limitations, cumulative distortion effects, or the re-use of links in the several combinations.

Figure 1 is a drafting perspective of the icosahedron with the vertices numbered from 1 to 12. Figure 2 is one planar redrawing, with the same numbering. It was discarded because of its apparent asymmetry, tending to overaccentuate points 1 and 2. In Fig. 3, the 12 points are arranged in two circles, inner and outer. A certain amount of experimenting with this configuration indicated that considerable redundancy could be added by using diagonals internal to the solid figure. This involves (1) a compromise of the number of diagonals added, and (2) some diagonals which are symmetrical in the solid figure are unsymmetrical in the planar representation. The diagonals selected (9 in number) are shown on Fig. 4. For Fig. 4, typical selected terminal combinations were analyzed; the results are shown in Table 2. Tables 3 through 7 list the particular paths which are summarized in Table 2. Each path is traced as a series of connected vertices.

TABLE 2

		Number of Combinations		
Terminals	Direct	2-Link	3-Link	4-Link
1 - 5	1	2	9	48
1 - 6	0	2	11	56
1 - 2	0	0	15	84
3 - 7	1	6	17	53
3 - 8	1	4	20	68

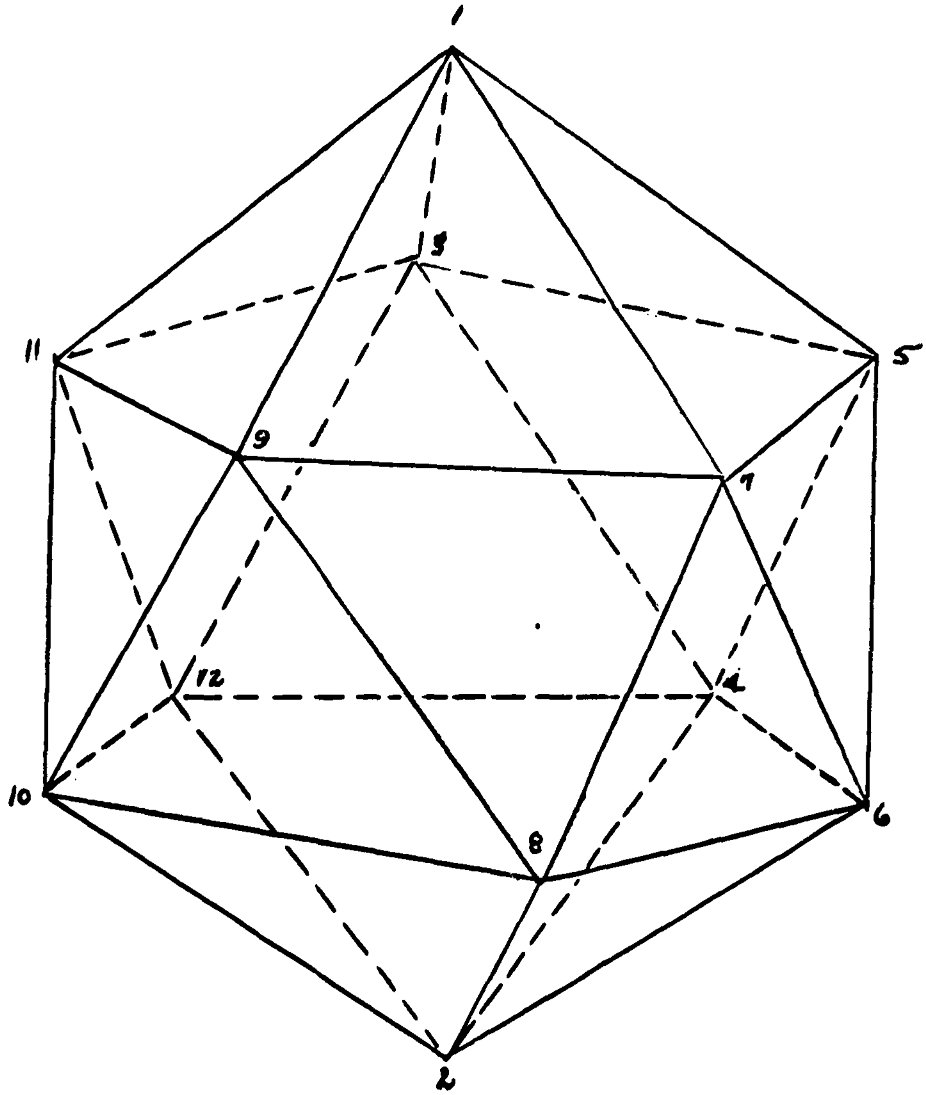

Fig. 1 Drafting Perspective of the Icosahedron

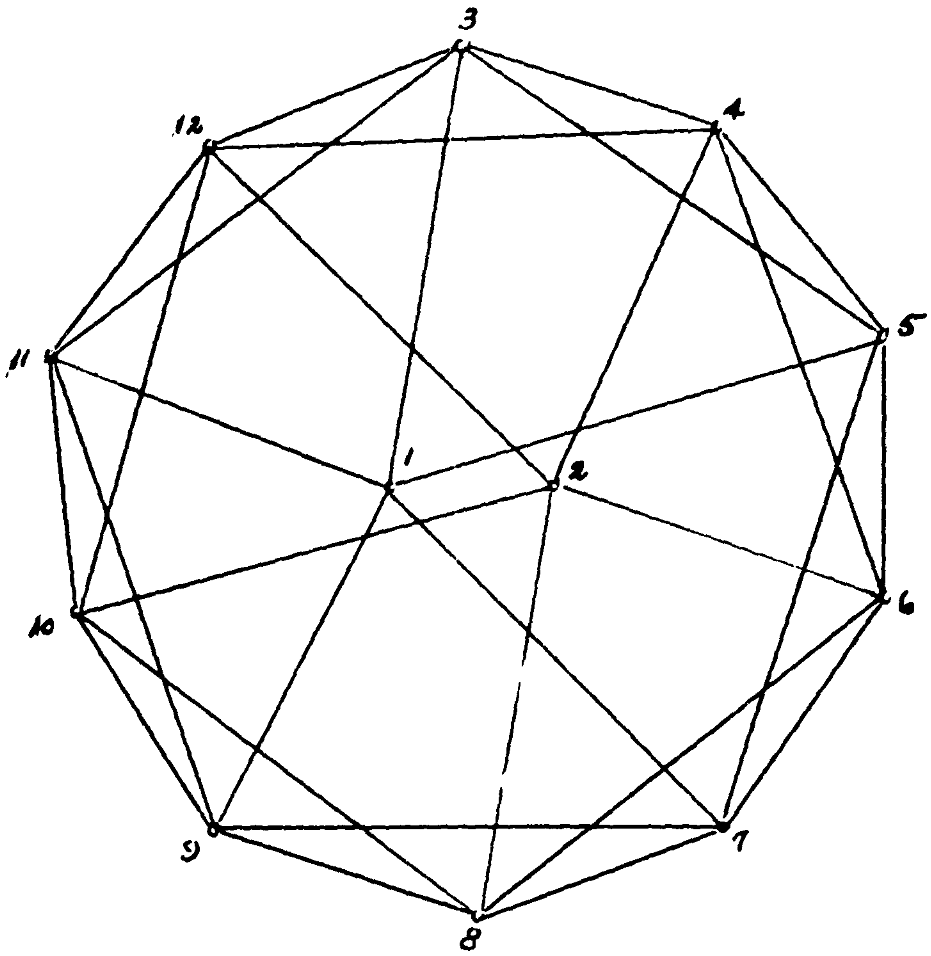

Fig. 2 Planar Version of the Icosahedron (Same Numbering)

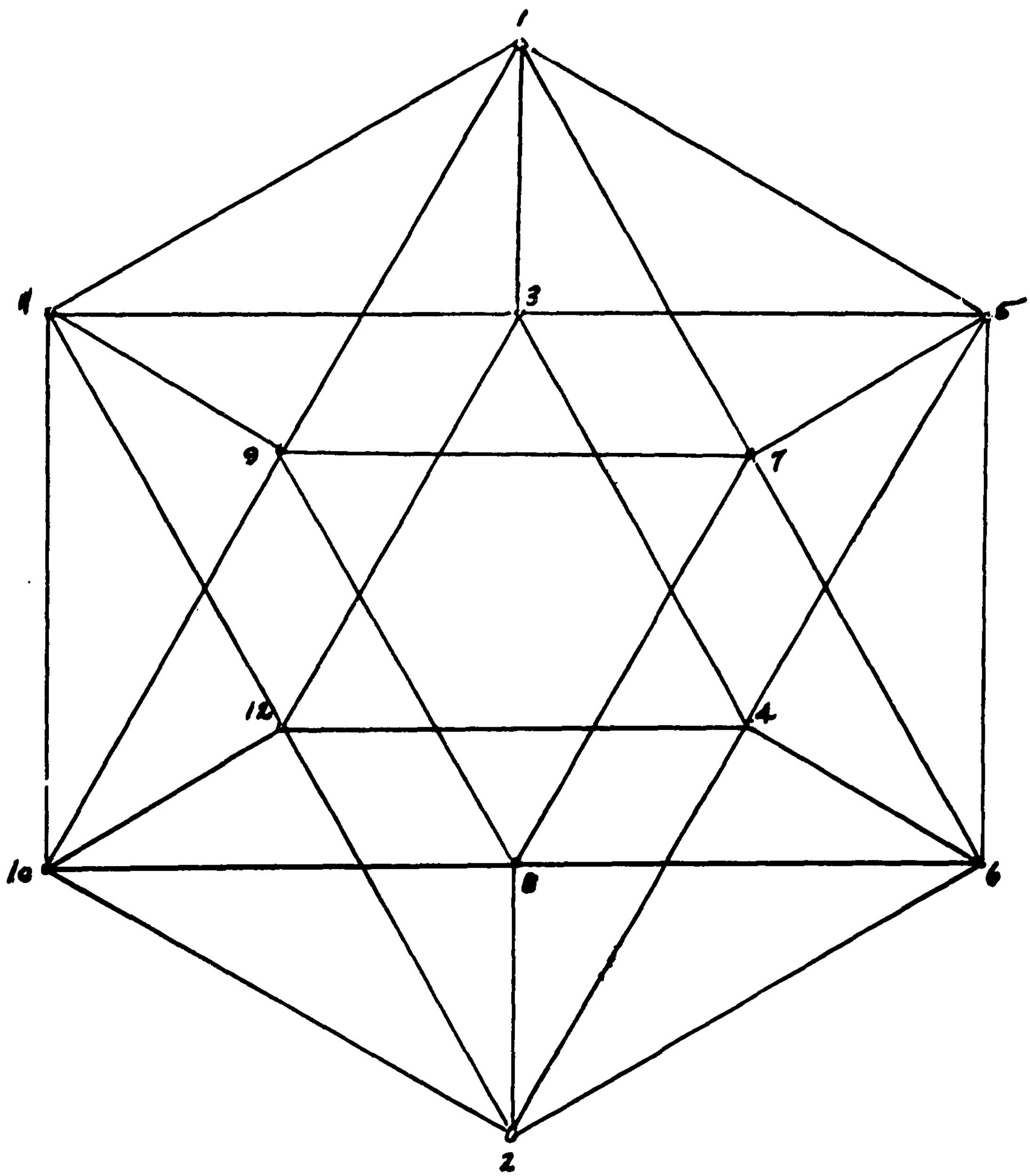

Fig. 3 Icosahedron With Rearranged Numbering System

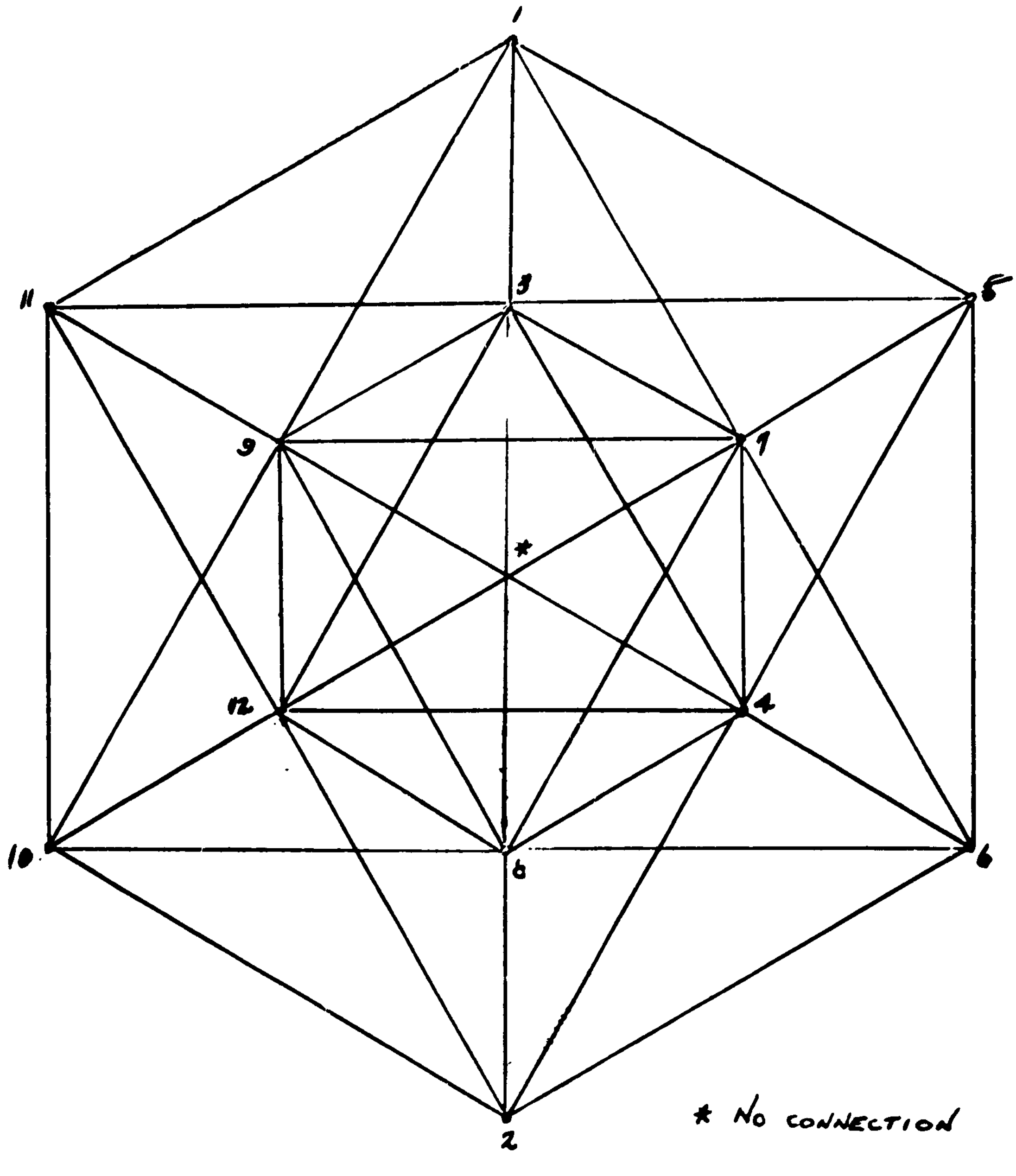

Fig. 4 Icosahedron With Nine Diagonals Added

TABLE 3

Icosahedron - Terminals 1 - 5

Direct Path	1	1-5	
2-Link	2	1-7-5	
3-Link	9	1-11-3-5	1-3-7-5
		1-9-3-5	1-3-4-5
		1-9-4-5	1-7-4-5
		1-9-7-5	1-7-3-5
			1-7-6-5
4-Link	48		

1-3-4-7-5	1-7-12-3-5	1-9-3-4-5	1-11-9-7-5
1-3-4-6-5	1-7-12-4-5	1-9-3-7-5	1-11-3-7-5
1-3-12-4-5	1-7-6-4-5	1-9-12-7-5	1-11-3-4-5
1-3-12-7-5	1-7-9-3-5	1-9-4-7-5	1-11-9-3-5
1-3-7-6-5	1-7-9-4-5	1-9-4-6-5	1-11-12-3-5
1-3-7-4-5	1-7-8-3-5	1-9-4-3-5	1-11-12-4-5
1-3-9-7-5	1-7-8-4-5	1-9-12-3-5	1-11-12-7-5
1-3-9-4-5	1-7-4-3-5	1-9-12-4-5	
1-3-8-7-5	1-7-4-6-5	1-9-11-3-5	
1-3-8-4-5	1-7-8-6-5	1-9-8-7-5	
1-3-8-6-5	1-7-3-4-5	1-9-8-4-5	
		1-9-8-6-5	
		1-9-7-4-5	
		1-9-7-6-5	
		1-9-8-3-5	
		1-9-8-6-5	
		1-9-7-3-5	
		1-9-3-4-5	

TABLE 4

Icosahedron - Terminals 1 - 6

Direct Path	0		
2-Link	2	1-5-6	
		1-7-6	
3-Link	11	1-3-5-6	1-5-4-6
		1-3-4-6	1-5-7-6
		1-3-7-6	1-9-4-6
		1-3-8-6	1-9-8-6
		1-7-5-6	1-9-7-6
		1-7-4-6	
A-Link	56		

1-3-5-4-6	1-5-4-2-6	1-7-5-4-6	1-9-3-7-6	1-11-3-5-6
1-3-7-5-6	1-5-4-8-6	1-7-4-5-6	1-9-3-5-6	1-11-3-7-6
1-3-7-4-6	1-5-4-7-6	1-7-4-2-6	1-9-3-4-6	1-11-3-8-6
1-3-7-8-6	1-5-7-4-6		1-9-3-8-6	1-11-9-4-6
1-3-4-5-6	1-5-7-8-6		1-9-7-5-6	1-11-12-2-6
1-3-4-2-6	1-5-3-7-6		1-9-7-4-6	1-11-12-8-6
1-3-8-4-6	1-5-3-4-6		1-9-7-8-6	1-11-12-4-6
1-3-8-2-6	1-5-3-8-6		1-9-4-5-6	1-11-12-7-6
1-3-8-7-6			1-9-4-8-6	1-11-10-2-6
1-3-12-8-6			1-9-4-2-6	1-11-10-8-6
1-3-12-4-6			1-9-4-7-6	
1-3-12-7-6			1-9-8-4-6	
1-3-12-2-6			1-9-8-7-6	
1-3-9-7-6			1-9-8-2-6	
1-3-9-4-6			1-9-12-4-6	
1-3-9-8-6			1-9-12-8-6	
			1-9-12-2-6	
			1-9-10-2-6	
			1-9-10-8-6	

TABLE 5

Icosahedron - Terminals 1 - 2

Direct Path 0

2-Link 0

3-Link 15

1-5-6-2	1-3-4-2
1-5-4-2	1-3-12-2
1-7-6-2	1-3-8-2
1-7-4-2	1-9-10-2
1-7-12-2	1-9-8-2
1-7-8-2	1-9-4-2
1-11-10-2	1-9-12-2
1-11-12-2	

4-Link 84

1-3-8-6-2	1-5-3-8-2	1-7-5-4-2	1-9-3-8-2	1-11-3-4-2
1-3-8-4-2	1-5-3-12-2	1-7-5-6-2	1-9-4-6-2	1-11-3-12-2
1-3-8-12-2	1-5-3-4-2	1-7-6-8-2	1-9-4-8-2	1-11-3-8-2
1-3-8-10-2	1-5-7-12-2	1-7-6-4-2	1-9-4-12-2	1-11-9-4-2
1-3-12-8-2	1-5-7-8-2	1-7-4-6-2	1-9-11-10-2	1-11-9-8-2
1-3-12-10-2	1-5-7-4-2	1-7-4-8-2	1-9-11-12-2	1-11-9-10-2
1-3-12-4-2	1-5-7-6-2	1-7-4-12-2	1-9-7-4-2	1-11-9-12-2
1-3-4-6-2	1-5-6-4-2	1-7-8-4-2	1-9-7-6-2	1-11-10-8-2
1-3-4-8-2	1-5-6-8-2	1-7-8-6-2	1-9-7-12-2	1-11-10-12-2
1-3-4-12-2	1-5-4-6-2	1-7-8-12-2	1-9-7-8-2	1-11-12-8-2
1-3-5-6-2	1-5-4-8-2	1-7-8-10-2	1-9-8-6-2	1-11-12-4-2
1-3-5-4-2	1-5-4-12-2	1-7-12-8-2	1-9-8-4-2	1-11-12-10-2
1-3-11-12-2		1-7-12-10-2	1-9-8-10-2	
1-3-11-10-2		1-7-12-4-2	1-9-8-12-2	
1-3-9-10-2		1-7-9-4-2	1-9-10-8-2	
1-3-9-12-2		1-7-9-8-2	1-9-10-12-2	
1-3-9-8-2		1-7-9-12-2	1-9-12-10-2	
1-3-9-4-2		1-7-9-10-2	1-9-12-8-2	
1-3-7-6-2		1-7-3-8-2	1-9-12-4-2	
1-3-7-4-2				
1-3-7-8-2				
1-3-7-12-2				

TABLE 6

Icosahedron Terminals 3 - 7

Direct Path	1	3-7	
2-Link	6	3-4-7	3-1-7
		3-5-7	3-9-7
		3-8-7	3-12-7
3-Link	17	3-1-5-7	3-8-4-7
		3-1-9-7	3-8-6-7
		3-4-6-7	3-9-4-7
		3-4-9-7	3-9-12-7
		3-4-8-7	3-11-9-7
		3-4-12-7	3-11-12-7
		3-5-6-7	3-12-8-7
		3-5-4-7	3-12-9-7
		3-5-1-7	
4-Link	53		

3-1-11-12-7	3-4-6-5-7	3-5-6-4-7	3-8-4-12-7	3-9-12-8-7	3-11-9-3-7
3-1-11-9-7	3-4-6-8-7	3-5-6-8-7	3-8-4-6-7	3-9-12-4-7	3-11-9-4-7
3-1-11-3-7	3-4-9-12-7	3-5-4-6-7	3-8-4-5-7	3-9-4-12-7	3-11-9-12-7
3-1-5-4-7	3-4-9-1-7	3-5-4-9-7	3-8-4-9-7	3-12-9-1-7	3-11-9-1-7
3-1-5-6-7	3-4-12-9-7	3-5-4-12-7	3-8-6-4-7	3-12-9-8-7	3-11-9-8-7
3-1-5-3-7	3-4-12-8-7	3-5-4-8-7	3-8-6-5-7	3-12-9-4-7	3-11-12-9-7
3-1-9-3-7	3-4-8-12-7	3-5-1-9-7			3-11-12-8-7
3-1-9-4-7	3-4-8-4-7				3-11-12-4-7
3-1-9-12-7	3-4-8-6-7				3-12-9-1-7
3-1-9-8-7	3-4-8-9-7				3-12-9-8-7
					3-12-9-4-7
					3-12-8-4-7
					3-12-8-6-7
					3-12-8-9-7

TABLE 7

Icosahedron Terminals 3 - 8

Direct Path	1	3-8	
2-Link	4	3-4-8	3-4-8
		3-7-8	3-12-8
3-Link	20	3-1-9-8	3-9-12-8
		3-1-7-8	3-9-10-8
		3-4-6-8	3-9-4-8
		3-4-12-8	3-9-7-8
		3-5-6-8	3-11-12-8
		3-5-4-8	3-11-9-8
		3-5-7-8	3-11-10-8
		3-7-4-8	3-12-4-8
		3-7-6-8	3-12-10-8
		3-7-12-8	
		3-7-9-8	
4-Link	68		

3-1-9-7-8	3-4-6-7-8	3-5-6-2-8	3-7-6-2-8	3-9-12-4-8	3-11-10-2-8
3-1-9-4-8	3-4-6-2-8	3-5-6-4-8	3-7-6-4-8	3-9-12-10-8	3-11-10-12-8
3-1-9-12-8	3-4-12-2-8	3-5-6-7-8	3-7-4-12-8	3-9-12-2-8	3-11-10-9-8
3-1-9-10-8	3-4-12-10-8	3-5-4-12-8	3-7-4-6-8	3-9-12-7-8	3-11-12-9-8
3-1-7-4-8	3-4-12-9-8	3-5-4-2-8	3-7-4-2-8	3-9-10-2-8	3-11-12-4-8
3-1-7-12-8		3-5-4-6-8	3-7-4-9-8	3-9-10-12-8	3-11-12-7-8
3-1-7-6-8		3-5-4-9-8	3-7-9-4-8	3-9-4-12-8	3-11-12-2-8
3-1-7-9-8		3-5-4-7-8	3-7-9-12-8	3-9-4-2-8	3-11-12-10-8
		3-5-7-6-8	3-7-9-10-8	3-9-4-7-8	3-11-9-10-8
		3-5-7-4-8	3-7-12-2-8	3-9-4-6-8	3-11-9-7-8
		3-5-7-12-8	3-7-12-4-8	3-9-7-12-8	3-11-9-12-8
		3-5-7-9-8	3-7-12-9-8	3-9-7-4-8	3-11-9-4-8
			3-7-12-10-8	3-9-7-6-8	
					3-12-10-2-8
					3-12-10-9-8
					3-12-4-7-8
					3-12-4-2-8
					3-12-4-6-8

The addition of more internal diagonals, up to a maximum of

$$\frac{11 \times 12}{2} - 30 = 36$$

would materially affect the number and distribution of paths.

The dodecahedron is shown in drafting perspective in Fig. 5. Figure 6 is a planar version of Fig. 5. It soon becomes obvious that more edges per vertex are required. Progressively, the diagram was converted to that of Fig. 7, which contains 18 switching points instead of 20 (for the dodecahedron) and has 7 connecting circuits per switching point. The switching points are arranged in inner and outer rings. Obviously, Fig. 7 does not represent a regular polyhedron.

A summary of the analysis made of the paths of Fig. 7 is given in Table 8.

TABLE 8

		Number of Combinations		
Terminals	Direct	2-Link	3-Link	4-Link
1 - 2	1	4	12	41
1 - 3	1	2	10	53
1 - 4	0	2	10	53
1 - 5	0	1	10	60
1 - 10	1	4	12	40
10 - 11	1	2	16	51
10 - 14	0	2	12	50

Tables 9 through 15 gives the particular paths which are summarized in Table 8.

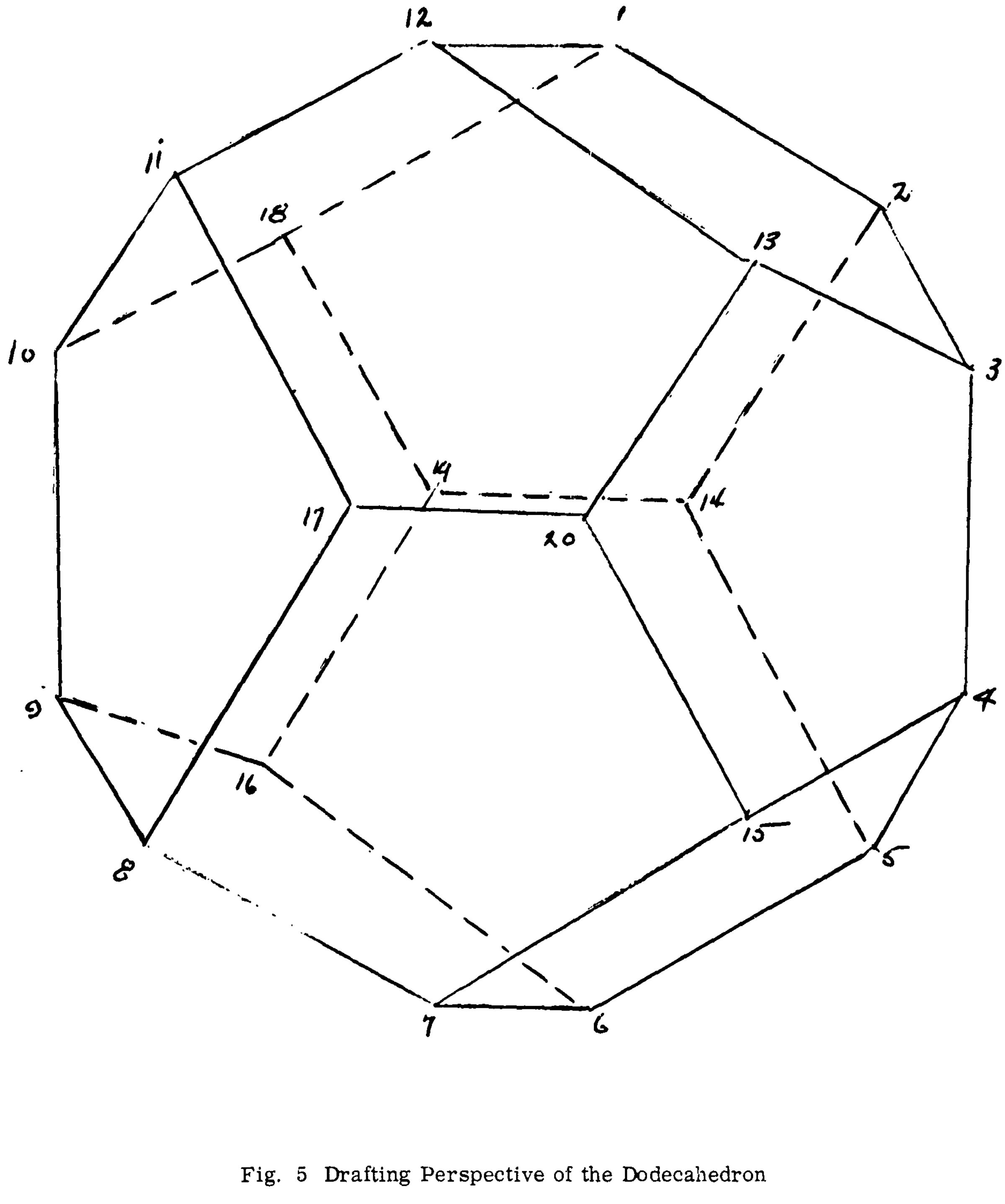

Fig. 5 Drafting Perspective of the Dodecahedron

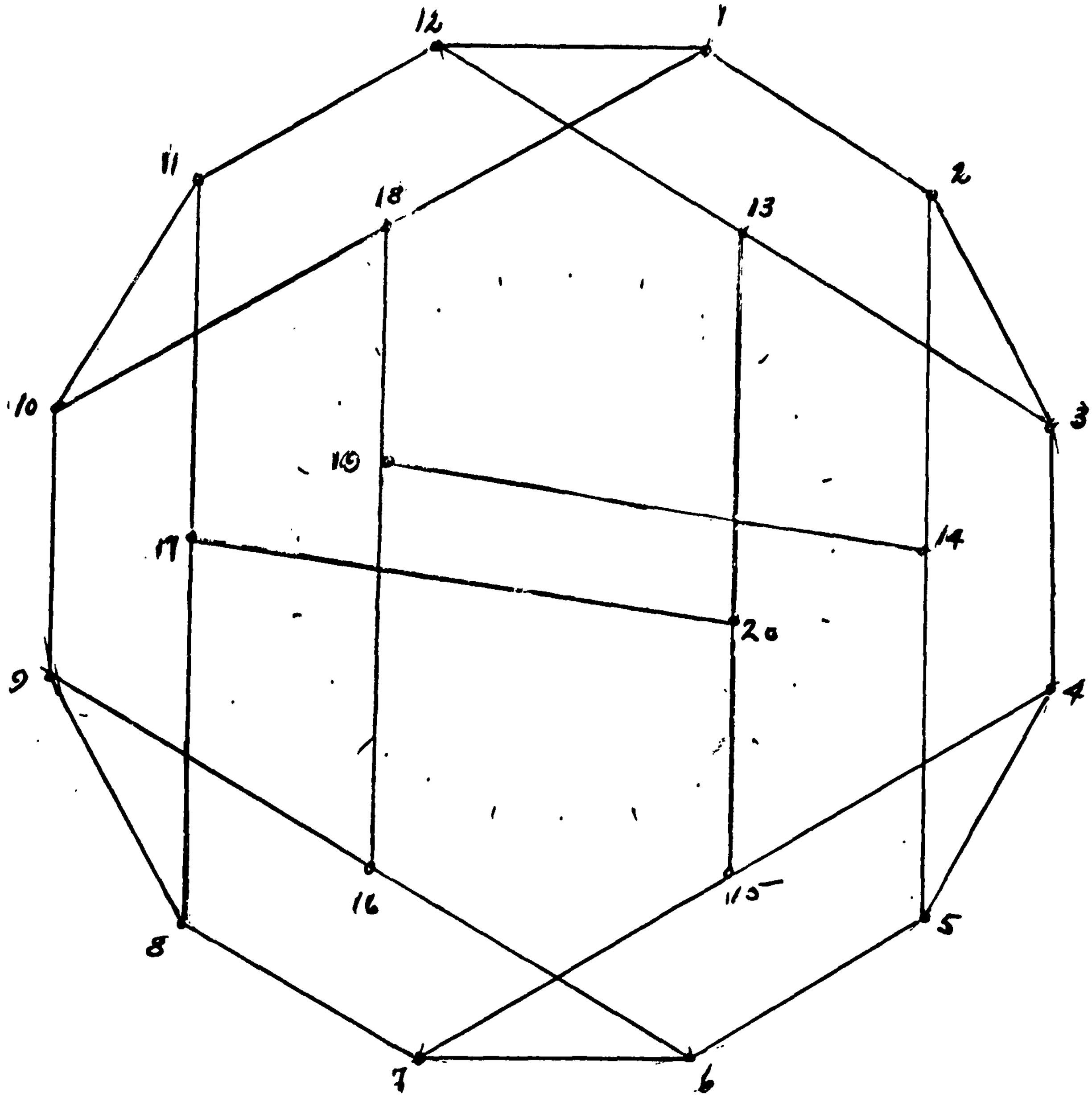

Fig. 6 Planar Version of the Dodecahedron

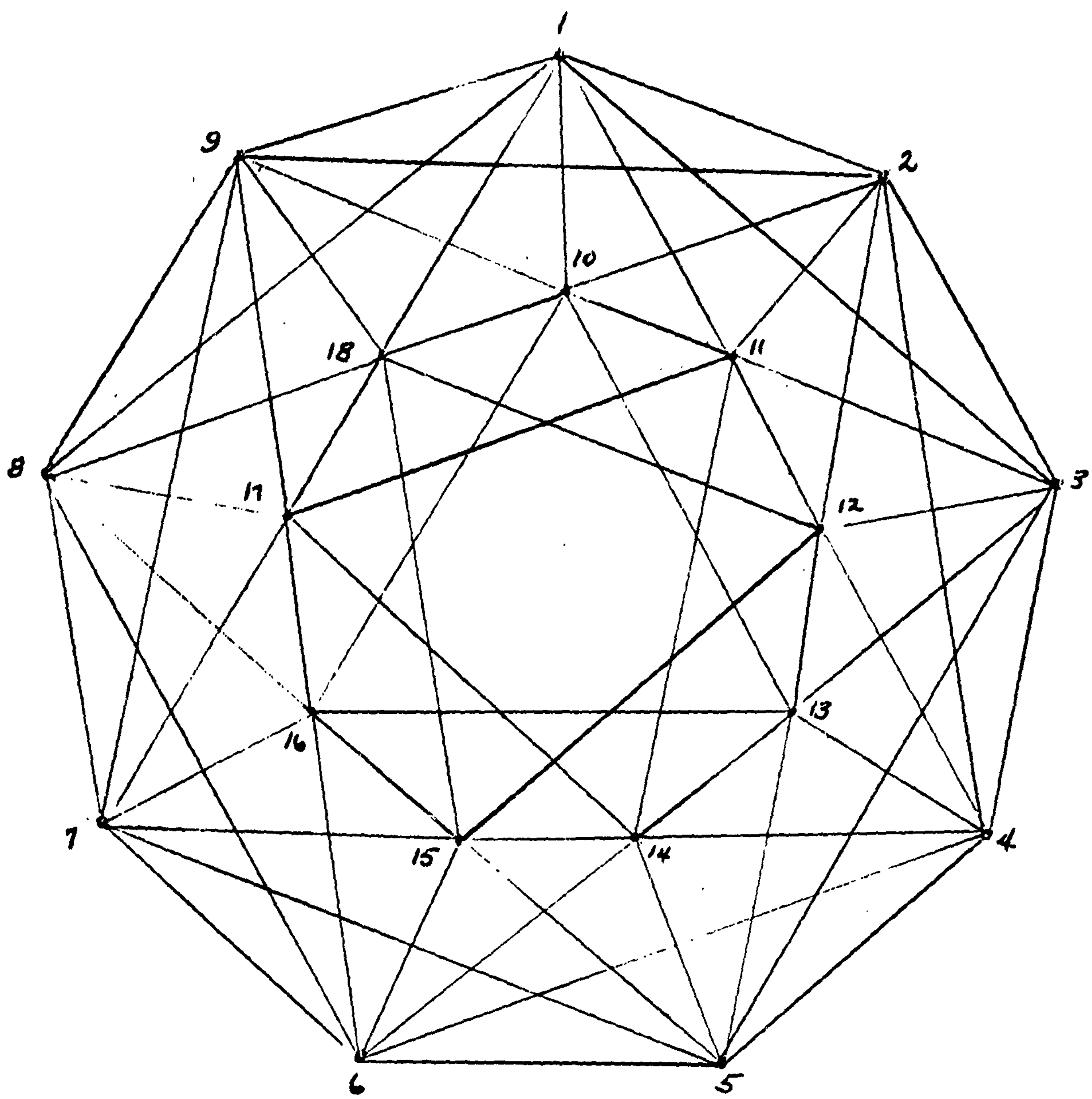

Fig. 7 Modified Version of Dodecahedron With 18 Switching Points

TABLE 9

Modified dodecahedron Terminals 1 - 2

Direct Path	1	1-2	
2-Link	4	1-10-2	
		1-11-2	
		1-3-2	
		1-9-2	
3-Link	12	1-9-10-2	1-8-9-2
		1-10-9-2	1-3-11-2
		1-10-11-2	1-3-12-2
		1-11-12-2	1-3-4-2
		1-11-3-2	1-18-10-2
		1-11-10-2	1-18-12-2
4-Link	41		

1-3-12-11-2	1-8-6-4-2	1-9-18-10-2	1-10-13-12-2
1-3-13-12-2	1-8-18-10-2	1-9-17-11-2	1-10-13-3-2
1-3-13-10-2	1-8-17-11-2	1-9-10-11-2	1-10-13-4-2
1-3-11-10-2	1-8-16-10-2	1-9-18-12-2	1-10-18-12-2
1-3-4-12-2	1-8-18-12-2		1-10-18-9-2
1-3-13-4-2	1-8-9-10-2		1-10-11-12-2
1-3-5-4-2	1-8-18-9-2		1-10-11-3-2
1-3-12-4-2			
1-11-12-3-2	1-18-17-9-2		
1-11-10-9-2	1-18-17-11-2		
1-11-17-9-2	1-18-15-12-2		
1-11-14-4-2	1-18-10-11-2		
1-11-12-4-2	1-18-12-11-2		
1-11-3-4-2	1-18-12-3-2		
	1-18-10-9-2		
	1-18-8-9-2		
	1-18-9-10-2		

TABLE 10

Modified dodecahedron Terminals 1 - 3

Direct Path	1	1-3	
2-Link	2	1-11-3 1-2-3	
3-Link	10	1-2-11-3 1-2-12-3 1-2-4-3 1-10-11-3 1-10-2-3 1-10-13-3	1-9-2-3 1-18-12-3 1-11-12-3 1-11-2-3
4-Link	53		

1-2-11-12-3	1-8-18-12-3	1-9-17-11-3	1-10-18-12-3
1-2-4-12-3	1-8-17-11-3	1-9-18-12-3	1-10-13-12-3
1-2-4-13-3	1-8-16-13-3	1-9-7-5-3	1-10-13-16-3
1-2-10-11-3	1-8-9-2-3	1-9-10-13-3	1-10-13-4-3
1-2-10-13-3	1-8-6-4-3	1-9-10-2-3	1-10-11-12-3
1-2-12-11-3	1-8-7-5-3	1-9-10-11-3	1-10-13-5-3
1-2-12-4-3	1-8-6-5-3	1-9-2-11-3	1-10-11-2-3
1-2-12-13-3		1-9-2-12-3	
		1-9-2-4-3	

1-11-2-4-3	1-18-12-4-3
1-11-2-12-3	1-18-12-11-3
1-11-12-13-3	1-18-12-13-3
1-11-12-4-3	1-18-12-4-3
1-11-12-2-3	1-18-12-2-3
1-11-10-2-3	1-18-10-11-3
1-11-10-13-3	1-18-10-2-3
1-11-14-4-3	1-18-10-13-3
1-11-14-13-3	1-18-15-5-3
1-11-14-5-3	1-18-15-12-3
	1-18-17-11-3
	1-18-9-2-3

TABLE 11

Modified dodecahedron Terminals 1 - 4

Direct Path	0		
2-Link	2	1-3-4	
		1-2-4	
3-Link	10	1-2-3-4	1-9-2-4
		1-2-12-4	1-11-12-4
		1-10-13-4	1-11-14-4
		1-8-2-4	1-11-2-4
		1-8-6-4	1-11-3-4
4-Link	53		

1-2-3-5-4	1-8-6-5-4	1-9-7-5-4	1-10-13-3-4
1-2-11-12-4	1-8-6-14-4	1-9-7-6-4	1-10-13-12-4
1-2-10-13-4	1-8-7-5-4	1-9-8-6-4	1-10-13-5-4
	1-8-7-6-4	1-9-2-3-4	1-10-13-14-4
	1-8-16-13-4	1-9-2-12-4	1-10-11-2-4
1-11-2-12-4	1-8-16-6-4	1-9-10-13-4	1-10-11-3-4
1-11-2-3-4		1-9-10-2-4	1-10-11-12-4
1-11-10-13-4		1-9-18-6-4	1-10-11-14-4
1-11-12-3-4	1-18-10-13-4	1-9-18-12-4	1-10-9-2-4
1-11-12-13-4	1-18-9-2-4		
1-11-3-5-4	1-18-17-14-4		
1-11-3-2-4	1-18-15-14-4		
1-11-3-13-4	1-18-15-5-4		
1-11-3-12-4	1-18-15-6-4		
1-11-14-5-4	1-18-8-6-4		
1-11-14-6-4	1-18-12-3-4		
1-11-14-13-4	1-18-12-13-4		
1-11-17-14-4	1-18-12-2-4		
	1-18-10-13-4		
	1-18-10-2-4		
	1-18-9-2-4		

TABLE 12

Modified dodecahedron Terminals 1 - 5

Direct path	0		
2-Link	1	1-3-5	
3-Link	10	1-2-4-5	1-10-13-5
		1-2-3-5	1-11-14-5
		1-3-4-5	1-11-3-5
		1-3-13-5	1-18-15-5
		1-8-6-5	
		1-9-7-5	
4-Link	60		

1-2-3-4-5	1-8-18-15-5	1-9-2-4-5	1-10-16-7-5	1-11-3-4-5
1-2-4-14-5	1-8-17-14-5	1-9-2-3-5	1-10-2-3-5	1-11-14-4-5
1-2-11-14-5	1-8-6-14-5	1-9-10-13-5	1-10-2-4-5	1-11-14-13-5
1-2-12-13-5	1-8-6-15-5	1-9-7-6-5	1-10-11-14-5	1-11-14-6-5
	1-8-6-4-5	1-9-7-15-5	1-10-11-3-5	1-11-14-15-5
	1-8-6-7-5	1-9-8-6-5	1-10-13-4-5	1-11-2-4-5
1-3-13-4-5	1-8-16-7-5	1-9-8-7-5	1-10-13-3-5	1-11-10-13-5
1-3-13-14-5	1-8-16-6-5	1-9-17-14-5	1-10-13-14-5	1-11-17-7-5
1-3-12-13-5	1-8-6-15-5	1-9-18-15-5	1-10-16-15-5	1-11-17-14-5
1-3-12-15-5	1-8-16-13-5		1-10-18-15-5	1-11-12-4-5
1-3-12-4-5	1-8-17-14-5		1-10-9-7-5	1-11-12-3-5
	1-8-17-7-5		1-10-16-6-5	1-11-12-15-5
	1-8-18-15-5		1-10-16-7-5	1-11-12-13-5
	1-8-9-7-5		1-10-16-13-5	
			1-10-16-15-5	

TABLE 13

Modified Dodecahedron Terminals 1 - 10

Direct path	1	1-10
2-Link	4	1-2-10 1-9-10 1-18-10 1-11-10
3-Link	12	1-2-11-10 1-2-9-10 1-3-2-10 1-3-11-10 1-3-13-10 1-8-16-10 1-8-9-10 1-8-18-10 1-9-2-10 1-9-18-10 1-11-2-10 1-18-9-10
4-Link	40	

1-2-3-13-10	1-3-2-9-10	1-8-17-11-10	1-9-17-11-10	1-11-17-18-10
1-2-3-11-10	1-3-2-11-10	1-8-16-13-10	1-9-2-11-10	1-11-17-9-10
1-2-4-13-10	1-3-4-2-10	1-8-7-9-10	1-9-7-16-10	1-11-2-9-10
1-2-9-18-10	1-3-4-13-10	1-8-9-18-10	1-9-17-16-10	1-11-12-18-10
1-2-12-18-10	1-3-12-11-10	1-8-7-16-10	1-9-8-16-10	1-11-3-13-10
1-2-12-13-10	1-3-13-16-10	1-8-9-2-10		1-11-17-16-10
	1-3-11-2-10	1-8-18-9-10		
	1-3-12-18-10	1-8-17-11-10		
		1-8-17-18-10		1-18-12-11-10
				1-18-12-2-10
				1-18-9-2-10
				1-18-17-11-10
				1-18-8-16-10
				1-18-12-13-10

TABLE 14

Modified dodecahedron Terminals 10 - 11

Direct path	1	10-11	
2-Link	2	10-1-11	
		10-2-11	
3-Link	16	10-1-2-11	10-13-12-11
		10-1-3-11	10-13-3-11
		10-1-18-11	10-13-14-11
		10-2-3-11	10-18-1-11
		10-2-1-11	10-18-17-11
		10-2-12-11	10-16-17-11
		10-9-2-11	10-18-12-11
		10-9-1-11	
		10-9-17-11	
4-Link	51		

10-1-9-2-11	10-2-3-12-11	10-9-8-17-11	10-13-3-2-11	10-16-8-1-11
10-1-3-12-11	10-2-1-3-11	10-9-8-1-11	10-13-4-2-11	10-16-8-17-11
10-1-3-2-11	10-2-4-12-11	10-9-1-2-11	10-13-4-12-11	10-16-13-12-11
10-1-2-3-11	10-2-12-3-11	10-9-1-3-11	10-13-14-13-11	10-16-15-12-11
10-1-2-12-11	10-2-4-3-11	10-9-2-3-11	10-13-16-17-11	10-16-15-14-11
10-1-9-17-11	10-2-3-1-11	10-9-2-12-11	10-13-5-14-11	10-16-13-3-11
10-1-9-2-11	10-18-15-12-11		10-13-3-1-11	10-16-13-14-11
10-1-18-12-11	10-18-15-14-11		10-13-4-3-11	10-16-6-14-11
10-1-8-17-11	10-18-9-2-11		10-13-12-2-11	10-16-7-17-11
	10-18-8-12-11		10-13-12-3-11	
	10-18-15-12-11			
	10-18-12-3-11			
	10-18-9-17-11			
	10-18-9-1-11			
	10-18-1-2-11			
	10-18-1-3-11			
	10-18-12-2-11			

TABLE 15

Modified dodecahedron Terminals 10 - 14

Direct path	0		
2-Link	2	10-11-14	
		10-13-14	
3-Link	12	10-18-15-14	10-16-13-14
		10-18-17-14	10-16-6-14
		10-1-11-14	10-2-4-14
		10-2-11-14	10-13-4-14
		10-9-17-14	10-11-17-14
		10-16-15-14	10-16-17-14
4-Link	50		

10-1-9-17-14	10-2-11-17-14	10-9-17-11-14	10-11-12-13-14
10-1-8-17-14	10-2-9-17-14	10-9-7-15-14	10-11-12-15-14
10-1-2-11-14	10-2-3-13-14	10-9-7-5-14	10-11-12-4-14
10-1-3-13-14	10-2-3-4-14	10-9-8-6-14	10-11-2-4-14
10-1-3-4-14	10-2-12-13-14	10-9-18-15-14	10-11-3-13-14
	10-2-9-17-14	10-9-2-11-14	10-11-3-4-14
10-13-3-4-14	10-16-15-6-14	10-18-15-5-14	
10-13-3-11-14	10-16-15-5-14	10-18-15-6-14	
10-13-16-15-14	10-16-6-4-14	10-18-12-13-14	
10-13-12-11-14	10-16-7-15-14	10-18-12-4-14	
10-13-12-15-14	10-16-7-6-14	10-18-8-6-14	
10-13-4-5-14	10-16-6-15-14	10-18-17-11-14	
10-13-3-5-14	10-16-6-5-14		
10-13-4-6-14	10-16-7-5-14		
	10-16-13-5-14		
	10-16-13-4-14		
	10-16-17-11-14		
	10-16-8-17-14		
	10-16-8-6-14		

An interesting comparison can be made between Figs. 4 and 7. Figure 4 has 6 vertices with 5 connections and 6 vertices with 8 connections. The number of connecting lines is

$$\frac{(6 \times 5) + (6 \times 8)}{2} = 39$$

This includes the 30 sides of the original polyhedron plus the 9 added diagonals. The <u>average</u> number of lines per switching point is $39/12 = 3.25$. Or the <u>average</u> number of connections per switching point is $2 \times 3.25 = 6.5$. More obviously for Fig. 7, the total number of lines is $7 \times 18/2 = 63$. The average number of lines per terminal is, of course, 3.5, which is not too different from the average of 3.25 noted above; the number of paths in the examples cited is also comparable.

The configuration of Fig. 7 appears to be a useful way of interconnecting inner and outer rings, and is probably worthy of further consideration.

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